

<b>MHF4U Course Outline</b>	
<b>Name of School</b>	Learning Languages Institute
<b>Department</b>	Mathematics
<b>Course Developer</b>	Shima Bimajal
<b>Course Development Date</b>	September 2017
<b>Course Reviser/Revision Date</b>	Hassan Mirzai
<b>Course Title</b>	Advanced Functions
<b>Grade</b>	12
<b>Course Type</b>	University/College
<b>Ministry Course Code</b>	MHF4U
<b>Credit Value</b>	1.0
<b>Developed From</b>	The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007 – Revised Growing Success (2010)
<b>Prerequisite</b>	Functions 11, MCR3U; or Mathematics for College Technology 12, MCT4C

Course Description:

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

## **Curriculum Overall Expectations**

<b>EXPONENTIAL AND LOGARITHMIC FUNCTIONS</b>	
A1	demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify numeric expressions;
A2	identify and describe some key features of the graphs of logarithmic functions, make connections among the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;

A3	solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.
<b>TRIGONOMETRIC FUNCTIONS</b>	
B1	demonstrate an understanding of the meaning and application of radian measure;
B2	make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals, and use these connections to solve problems;
B3	solve problems involving trigonometric equations and prove trigonometric identities.
<b>POLYNOMIAL AND RATIONAL FUNCTIONS</b>	
C1	identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;
C2	identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;

C3	solve problems involving polynomial and simple rational equations graphically and algebraically;
C4	demonstrate an understanding of solving polynomial and simple rational inequalities.
<b>CHARACTERISTICS OF FUNCTIONS</b>	
D1	demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;
D2	determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;
D3	compare the characteristics of functions, and solve problems by modelling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

A	EXPONENTIAL AND LOGARITHMIC FUNCTIONS
B	TRIGONOMETRIC FUNCTIONS

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C	POLYNOMIAL AND RATIONAL FUNCTIONS
D	CHARACTERISTICS OF FUNCTIONS

## Units Listed in the Sequence of Delivery

Unit	Titles and Descriptions	Time and Sequence
Unit 1	<p><b>Functions: Characteristics and Properties</b></p> <p>Many situations can be modelled graphically. Interpreting those graphs is something that requires you to become familiar with all of the aspects of these graphs. Students will recall a few basic facts of a distance time graph. In previous math courses, students saw some transformations and studied their effects on a given graph. These will also be reviewed. The concepts of function notation, relation, range, domain, and function notation will all be reviewed.</p>	10 hours

Unit 2	<p><b>Polynomial Functions</b></p> <p>In this unit students learn to identify and describe some key features of polynomial functions and to make connections between the numeric, graphical, and</p>	10 hours
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	algebraic representations of polynomial functions. Transformations of Cubic and Quartic functions. Dividing and Factoring Polynomials.	
Unit 3	<b>Polynomial Equations and Inequalities</b> Solving polynomial equations. Solving linear and polynomial equations by using factoring and graphing.	13 hours
Unit 4	<b>Rational Functions, Equations and Inequalities</b> The concepts of identifying and describing some key features of the graphs of rational functions are also presented. Students will represent rational functions graphically; solve problems involving polynomial and simple rational equations graphically and algebraically so they can demonstrate an understanding of how to solve rational inequalities.	13 hours
Unit 5	<b>Trigonometric Functions</b> This unit examines the meaning and application of radian measure. Students will make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals. Transformations of Trigonometric Functions	15 hours
Unit 6	<b>Trigonometric Identities and Equations</b>	13 hours

	Exploring equivalent trigonometric functions. Compound and Double angle formulas. Proving Trigonometric Identities. Solving linear and quadratic trigonometric equations.	
Unit 7	<p><b>Exponential and Logarithmic Functions</b></p> <p>This unit begins with a review of the rules associated with exponents. Exponential functions, examples and applications of them are the first new topic in this unit followed by logarithmic functions, identities and applications. Solving Exponential and Logarithmic equations.</p>	15 hours
Unit 8	<p><b>Functions: Understanding Rates of Change</b></p> <p>Determining Average rate of change, estimating Instantaneous rate of change from tables of values and equations. Exploring Instantaneous rate of change using graphs. Using rates of change to create a graphical model. Solving Problems involving rates of change.</p>	11 hours
Unit 9	<p><b>Combinations of Functions</b></p> <p>Focuses on the theory and practice of performing arithmetic operations on entire functions, including but not limited to the algebraic, graphical and practical implications of performing those operations. Sums, Differences, Products and Quotients of Functions. Composition of Functions</p>	10 hours

Final Evaluation 30% of your grade will be determined at the end of the course.		
Total	110 hours	

## Teaching and Learning Strategies

A wide variety of instructional strategies are used to provide learning opportunities to accommodate a variety of learning styles, interests and ability levels. These strategies include, but are not limited to:

- Think-Pair-Share
- Concept Attainment
- Peer Assessment
- Entry Slip
- Model Analysis
- Group Discussion
- Anticipation Guide
- Problem Solving
- Graphing Applications
- Self-Assessments



- Investigate
- KWL Chart
- Research
- Direct Instruction
- Graphic Organizers
- Exit card
- *Connecting*: Activating prior knowledge when introducing a new concept in order to make a smooth connection between previous learning and new concepts, and introducing skills in context to make connections between particular manipulations and problems that require them.
- *Reasoning and proving*: Asking questions that get students to hypothesize, providing students with one or more numerical examples that parallel these with the generalization and describing their thinking in more detail.
- *Problem solving*: Scaffolding of knowledge, detecting patterns, making and justifying conjectures, guiding students as they apply their chosen strategy, directing students to use multiple strategies to solve the same problem, when appropriate, recognizing, encouraging, and applauding perseverance, discussing the relative merits of different strategies for specific types of problems.
- *Communicating*: To improve student success there will be several opportunities for students to share their understanding both in oral as well as written form

## Strategies for Assessment and Evaluation of Student Performance

	Unit	Time	Expectation	AFL	AAL	AOL	K 25%	A 25%	C 25%	T 25%
70% Term Work	A	25 Hours	A1-A3	Diagnostic Assessment Home Work Unit Review Practice Test Exit card	Reflective Discussion On Learning	Unit Assignment Test Presentation	*	*	*	*
	B	28 Hours	B1-B3	Diagnostic Assessment Assigned Questions Unit Wrap Up Exit card	KWL Chart	Unit Assignment Test Presentation	*	*	*	*
	C	36 Hours	C1-C4	Prior Knowledge Assessment Worksheet Unit Review Unit Wrap Up Discussion Exit card	KWL Chart	Unit Assignment Test Presentation	*	*	*	*

	D	21 Hours	D1-D3	Home Work Unit Review Unit Wrap Up Exit card	Pair Peer Assessme nt	Assignment Presentation Test	*	*	*	*
30% Summ ative		3 Hours	A1-D3			Final Exam	*	*	*	*

## Assessment and Evaluation Strategies for MHF4U

### Learning Skills & Work Habits:

*These learning skills and work habits will be taught, assessed and evaluated throughout the course.*

<p>Responsibility</p> <ul style="list-style-type: none"> <li>● Fulfils responsibilities and commitments within the learning environment</li> <li>● Completes and submits class work, homework, and assignments according to agreed-upon timelines.</li> </ul>	<p>Collaboration</p> <ul style="list-style-type: none"> <li>● Accepts various roles and an equitable share of work in a group</li> <li>● Responds positively to the ideas, opinions, values, and traditions of others</li> <li>● Builds healthy peer-to-peer relationships through personal and media-assisted interactions</li> </ul>
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<ul style="list-style-type: none"> <li>● Takes responsibility for and manages own behavior</li> </ul>	<ul style="list-style-type: none"> <li>● Works with others to resolve conflicts and build consensus to achieve group goals</li> <li>● Shares information, resources, and expertise, and promotes critical thinking to solve problems and make decisions</li> </ul>
<p>Organization</p> <ul style="list-style-type: none"> <li>● Devises and follows a plan and process for completing work and tasks</li> <li>● Establishes priorities and manages time to complete tasks and meet goals</li> <li>● Identifies, gathers, evaluates, and uses information, technology, and resources to complete tasks</li> </ul>	<p>Self- Regulation</p> <ul style="list-style-type: none"> <li>● Sets own individual goals and monitors progress towards achieving them</li> <li>● Seeks clarification or assistance when needed</li> <li>● Assesses and reflects critically on own strengths, needs and interests</li> <li>● Identifies learning opportunities, choices, and strategies to meet personal needs and meet goals</li> <li>● Perseveres and makes an effort when responding to challenges</li> </ul>

<p>Independent Work</p> <ul style="list-style-type: none"> <li>● Independently monitors, assesses, and revises plans to complete tasks and meet goals</li> <li>● Uses class time appropriately to complete tasks</li> <li>● Follows instructions with minimal supervision</li> </ul>	<p>Initiative</p> <ul style="list-style-type: none"> <li>● Looks for and acts on new ideas and opportunities for learning</li> <li>● Demonstrates the capacity for innovation and a willingness to take risks</li> <li>● Demonstrates curiosity and interest in learning</li> <li>● Approaches new tasks with a positive attitude</li> <li>● Recognizes and advocates appropriately for the rights of self and others</li> </ul>
<p>E – Excellent G – Good S – Satisfactory N – Needs Improvement</p>	

**Course Policies:**

Missed Tests and Quizzes:

- If you miss a test or quiz, you will be expected to write it on your first day back, unless other arrangements are made in advance.
- Accommodations will be made for school activities, statutory holidays, religious days, cultural days and other occurrences that may impact on any

scheduled evaluation. It is the student's responsibility to notify teachers of such absences in advance and to make up missed work.

- Absence on the day of an evaluation must be documented (ex. Doctor's note).  
If a student must miss an evaluation, s/he is expected to:
  - a. see the teacher before the absence to arrange for an alternative date to make up the evaluation; or
  - b. in case of illness or unexpected absence, present a note to the teacher, signed by a parent or guardian, immediately upon their return to explain the absence. An alternate evaluation will then be scheduled.

**Absences:**

- If you are absent, it is your responsibility to catch up on the material that was covered during the day(s) that you were absent. It'd be helpful to know your classmates in order to ask for missed homework and material covered.
- If you miss more than 50% of a class period, you will be marked absent.

**Late assignment:**

- You are responsible for handing in material within the time frame specified by the teacher and in a form approved by the teacher. Late submission, without an acceptable reason, will result in 5% deduction per day.

Evaluation	Break Down
Tests	40 %

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Assignments	20 %
Final Presentation	10%
Final exam	30 %

KICA Breakdown:

Knowledge/ Understanding: 30%

Application/ Making Connection: 30%

Thinking: 20%

Communication: 20%

## Consideration for Program Planning

When planning a program in mathematics, teachers must take into account considerations in a number of important areas, including those discussed below.

### PLANNING MATHEMATICS PROGRAMS FOR STUDENTS WITH SPECIAL EDUCATION NEEDS

Classroom teachers are the key educators of students who have special education needs. They have a responsibility to help all students learn, and they work collaboratively with special education teachers, where appropriate, to achieve this goal. Special Education Transformation: The Report of the Co-Chairs with the Recommendations of the Working Table on Special Education, 2006 endorses a set of beliefs that should guide program planning for students with special education needs in all disciplines. Those beliefs are as follows: All students can succeed.

Universal design and differentiated instruction are effective and interconnected means of meeting the learning or productivity needs of any group of students. Successful instructional practices are founded on evidence-based research, tempered by experience. Classroom teachers are key educators for a student's literacy and numeracy development. Each student has his or her own unique patterns of learning. Classroom teachers need the support of the larger community to create a learning environment that supports students with special education needs. Fairness is not sameness. In any given classroom, students may demonstrate a wide range of learning styles and needs. Teachers plan programs that recognize this diversity and give students performance tasks that respect their particular abilities so that all students can derive the greatest possible benefit from the teaching and learning process. The use of flexible groupings for instruction and the provision of ongoing assessment are important elements of programs that accommodate a diversity of learning needs.

## **PROGRAM CONSIDERATIONS FOR ENGLISH LANGUAGE LEARNERS**

Ontario schools have some of the most multilingual student populations in the world. Ontario's linguistic heritage includes several Aboriginal languages and many African, Asian, and European language.

When these students start school in Ontario, they are entering a new linguistic and cultural environment. All teachers share in the responsibility for these students' English language development.



English language learners (students who are learning English as a second or additional language in English-language schools) bring a rich diversity of background knowledge and experience to the classroom. These students' linguistic and cultural backgrounds not only support their learning in their new environment but also become a cultural asset in the classroom community. Teachers will find positive ways to incorporate this diversity into their instructional programs and into the classroom environment.

Most English language learners in Ontario schools have an age-appropriate proficiency in their first language. Although they need frequent opportunities to use English at school, there are important educational and social benefits associated with continued development of their first language while they are learning English. Teachers need to encourage parents to continue to use their own language at home in rich and varied ways as a foundation for language and literacy development in English. It is also important for teachers to find opportunities to bring students' languages into the classroom, using parents and community members as a resource. During their first few years in Ontario schools, English language learners may receive support through one of two distinct programs from teachers who specialize in meeting their language-learning needs:

In planning programs for students with linguistic backgrounds other than English, teachers need to recognize the importance of the orientation process, understanding that every learner needs to adjust to the new social environment and language in a unique way and at an individual pace. For example, students who are in an early stage of English-language acquisition may go through a "silent period" during which they closely observe the interactions and physical surroundings of their new learning environment. They may use body language rather than speech or they may use their

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first language until they have gained enough proficiency in English to feel confident of their interpretations and responses. Students thrive in a safe, supportive, and welcoming environment that nurtures their self-confidence while they are receiving focused literacy instruction. When they are ready to participate in paired, small-group, or whole-class activities, some students will begin by using a single word or phrase to communicate a thought, while others will speak quite fluently.

Research has shown that it takes five to seven years for most English language learners to catch up to their English-speaking peers in their ability to use English for academic purposes. Moreover, the older the children are when they arrive, the more language knowledge and skills they have to catch up on, and the more direct support they require from their teachers.

Responsibility for students' English-language development is shared by the classroom teacher, the ESL/ELD teacher (where available), and other school staff. Volunteers and peers may also be helpful in supporting English language learners in the language classroom.

Teachers must adapt the instructional program in order to facilitate the success of these students in their classrooms. Appropriate adaptations include:

- modification of some or all of the subject expectations so that they are challenging but attainable for the learner at his or her present level of English proficiency, given the necessary support from the teacher;
- use of a variety of instructional strategies (e.g., extensive use of visual cues, graphic organizers, and scaffolding; previewing of textbooks; pre-teaching of key vocabulary; peer tutoring; strategic use of students' first languages);
- use of a variety of learning resources (e.g., visual material, simplified text, bilingual

dictionaries, and materials that reflect cultural diversity);

- use of assessment accommodations (e.g., granting of extra time; use of oral interviews, demonstrations or visual representations, or tasks requiring completion of graphic organizers or cloze sentences instead of essay questions and other assessment tasks that depend heavily on proficiency in English).

When learning expectations in any course are modified for an English language learner (whether the student is enrolled in an ESL or ELD course or not), this information must be clearly indicated on the student's report card.

## **ANTIDISCRIMINATION EDUCATION**

The implementation of antidiscrimination principles in education influences all aspects of school life. It promotes a school climate that encourages all students to work to attain high standards, affirms the worth of all students, and helps students strengthen their sense of identity and develop a positive self-image. It encourages staff and students alike to value and show respect for diversity in the school and the wider society. It requires schools to adopt measures to provide a safe environment for learning, free from harassment, violence, and expressions of hate.

Antidiscrimination education encourages students to think critically about themselves and others in the world around them in order to promote fairness, healthy relationships, and active, responsible citizenship.

Access to computers should be monitored and a range of software applications provided.

A problem-solving approach can benefit students who are having difficulties with materials or equipment. Because access to equipment at home will vary, it is important to offer challenges for or support to students whose levels of prior knowledge differ.

Critical thinking is the process of thinking about ideas or situations in order to understand them fully, identify their implications, and/or make a judgement about what is sensible or reasonable to believe or do. Critical thinking includes skills such as questioning, predicting, hypothesizing, analyzing, synthesizing, examining opinions, identifying values and issues, detecting bias, and distinguishing between alternatives.

## **LITERACY, MATHEMATICAL LITERACY, AND INVESTIGATION (INQUIRY/RESEARCH) SKILLS**

Literacy, mathematical literacy, and investigation skills are critical to students' success in all subjects of the curriculum and in all areas of their lives. Many of the activities and tasks that students undertake in the science curriculum involve the literacy skills related to oral, written, and visual communication. Communication skills are fundamental to the development of scientific literacy, and fostering students' communication skills is an important part of the teacher's role in the mathematics curriculum.

To develop their oral communication skills, students need numerous opportunities to listen to information and talk about a range of subjects in mathematics. The science program provides opportunities for students to engage in various oral activities in connection with expectations in all the strands, such as brainstorming to identify what they know about the new topic they are studying, discussing strategies for solving a problem, presenting and defending ideas or debating issues, and offering critiques of models and results produced by their peers.

## **THE ROLE OF INFORMATION AND COMMUNICATION TECHNOLOGY IN MATHEMATICS**

Information and communication technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' learning in mathematics. Teachers can use ICT tools and resources both for whole-class instruction and to design programs that meet diverse student needs. Technology can help to reduce the time spent on routine mathematical tasks, allowing students to devote more of their efforts to thinking and concept development. Useful ICT tools include simulations, multimedia resources, databases, sites that give access to large amounts of statistical data, and computer-assisted learning modules. Applications such as databases, spreadsheets, dynamic geometry software, dynamic statistical software, graphing software, computer algebra systems (CAS), word-processing software, and presentation software can be used to support various methods of inquiry in mathematics. Technology also makes possible simulations of complex systems that can be useful for problem-solving purposes or when field studies on a particular topic are not feasible. Information and

communications technologies can be used in the classroom to connect students to other schools, at home and abroad, and to bring the global community into the local classroom. Although the Internet is a powerful electronic learning tool, there are potential risks attached to its use. All students must be made aware of issues of Internet privacy, safety, and responsible use, as well as of the ways in which this technology is being abused – for example, when it is used to promote hatred. Teachers, too, will find the various ICT tools useful in their teaching practice, both for whole class instruction and for the design of curriculum units that contain varied approaches to learning to meet diverse student needs.

## **CAREER EDUCATION IN MATHEMATICS**

Teachers can promote students' awareness of careers involving mathematics by exploring applications of concepts and providing opportunities for career-related project work. Such activities allow students the opportunity to investigate mathematics-related careers compatible with their interests, aspirations, and abilities. Students should be made aware that mathematical literacy and problem solving are valuable assets in an ever-widening range of jobs and careers in today's society. The knowledge and skills students acquire in mathematics courses are useful in fields such as science, business, engineering, and computer studies; in the hospitality, recreation, and tourism industries; and in the technical trades.

### **Resources:**

- Growing Success Documents, Ministry of Education, 2010.
- Nelson Advanced Functions, Nelson Education Ltd. © 2009.

- <https://www.kutasoftware.com/>
- <https://www.everythingmaths.co.za/read/maths/grade-12>